Not exactly: a challenge for the QUD-based approach to imprecision

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1 Introduction

This talk is about the imprecise (loose) readings of numerical expressions. Consider some examples:

(1) A high school principal in Texas travelled **800 miles** in two weeks to visit all of his 612 graduating seniors at home.
(https://www.businessinsider.com.au/texas-principal-visited-every-graduating-senior-2020-4)

- '800 miles' is supposed to be read imprecisely (not intended to mean 800 miles to the nearest nanometer); but '612' is supposed to be read exactly.

(2) **66 million years old**, **4 metres high**, **12 metres long** and with almost all its terrifying teeth intact: Tristan Otto, the king of the dinosaurs, is in Copenhagen. (https://snm.ku.dk/english/exhibitions/tristan_otto/)

- All three numerical expressions are supposed to be read imprecisely. But we can allow a higher margin of error for '66 million' than for '4' and '12'.

(3) Tampa is **exactly 1000 miles** away from New York city.

- 'Exactly 1000 miles' probably means 1000 miles to the nearest mile rather than the nearest nanometer. So it is also supposed to be read imprecisely.

(4) **I am interested in these questions:**
   a. What are the denotations of numerical expressions? Points or intervals? Call the former view *pointism* and the latter view *intervalism*.
   b. What are the pros and cons of pointism and intervalism?
   c. What do the modifiers (slack regulators (Lasersohn 1999)) 'exactly' and 'about' do?

(5) **The goals of this talk:**
   a. Consider an argument against pointism
   b. Sketch what an intervalist account should look like.

Before I do those, I need to lay out the pointist's view.

2 Pointism

In the following, we will focus on the following two sentences, i.e. (C) and (EC), and their negations:

(C) (Stephen) Curry is 190 cm (based on multiple sources on the internet)

(EC) Curry is exactly 190 cm

(¬C) Curry is not 190 cm

(¬EC) Curry is not exactly 190 cm

2.1 Commitments and motivations

(6) **Pointism's commitments:**

a. The denotations of numerical expressions are points and so the semantic content (literal content/ what is said) of (C) is false

b. The **relevance maxim ('be relevant')** plays a crucial role in explaining why we hear (C) as a true sentence.

- Notable pointists: Lasersohn (1999), Klecha (2018), Hoek (2018), Carter (2017, 2021)

(7) **Motivations of pointism:**

a. **Avoids (postulating) ambiguity** ('190 cm' only has one meaning, the strict/ precise/ intolerant one; not both a strict meaning and many loose meanings)

b. **Avoids arbitrary selection of interval** (natural for the intervalist to say that '190 cm' denote different intervals in different contexts. But which one in which context?)

c. **Parsimony**: If the relevance maxim together with the strict meanings of numerical expressions can account for the imprecise readings of numerical expressions, it does seem to be more parsimonious than a theory that accepts both strict meanings and loose meanings (other things being equal).

2.2 Relevance, question under discussion (QUD), and content calibration

Recent pointist accounts (Klecha 2018, Hoek 2018) use the technical concept of *Question under Discussion (QUD)* (Roberts 2012) to (i) make explicit what it is for a piece of information to be relevant and (ii) calibrate the contents of sentences like (C) into intuitive true contents.

(8) **What is a QUD?**
A QUD determines what is currently *relevant* in a conversation, allowing the interlocutors to focus on one aspect of the world in their joint inquiry into what the world is like.

For example, if you are interested in finding out who won the 3-point shooting contest this year, you can guide your conversation with me by raising the question 'Who won the 3-point shooting context this year?'. If I accept your question, as a cooperative interlocutor, I am required to help you choose between the relevant answers to it (e.g. Curry, Lavine, Tatum, Mitchell), or at least help you narrow down the relevant answers to it (e.g. Curry or Lavine).

(9) A QUD need not be raised explicitly by asking a question. We call a QUD that isn't explicitly raised an **implicit** QUD.

For example, suppose a mum saw that her vase was broken and tried to find out which child of hers did it by glaring at all her children. One of her children can sensibly utter 'not me' even though her mum did not raise the QUD *Who broke my vase?* explicitly.

(10) Formally, a QUD is a partition over the context set (Stalnaker 1978).

For example, if the information that are mutually accepted among us is represented by the set of worlds in the rectangle below, the QUD *Who won the 3-point contest this year?* will divide those worlds into cells, each of which represents a complete answer to the question.

(11)

(12)

Curry	Tatum
Lavine	Mitchell

(Assumption: there are only 4 players)

In the top left cell are worlds in which Curry won the 3-point contest this year. In the top right cell are worlds in which Tatum won the context, and so on.

(13) **Why is QUD useful to the pointist?**
In everyday conversations, the difference between 190.0 cm and 190.01 cm is almost never relevant to whether 'Curry is 190 cm' counts as true. The pointist can explain this phenomenon by positing an implicit QUD such as *What is Curry's height to the nearest 5cm?*.

The thought is that if someone utters (C), and if the hearer doesn't object to their utterance, then the QUD *What is Curry's height to the nearest 5cm?* will be tacitly accepted by the hearer, and

that QUD will calibrate the content of (C) into an answer to the QUD. For example, consider the following representation of the QUD *What is Curry's height to the nearest 5cm?*:

(14)

180+/-5	200+/-5
190+/-5 x	210+/-5

*top left: worlds in which Curry's height falls into the interval [175 cm,185 cm]
*blue cross: the worlds in which Curry's height is exactly 190 cm (to the nearest nanometer); also the semantic content of (C)

Since the semantic content of (C) lies within the bottom left cell, it signals to the hearer that the answer to the question is the bottom left cell, which is the proposition that Curry's height is within [185,195]. This is the pointist's account of how the content of (C) gets calibrated by the QUD.

2.3 Content calibration for negations and sentences with 'exactly'

2.3.1 Negations

We turn now to how the pointists use QUD to coarsen the contents of negations and of sentences with slack regulators such as 'exactly'. Consider the negation of C:

(¬C) Curry is not 190 cm.

The semantic content of (C) is represented by the blue cross below. This means that the semantic content of (¬C) overlaps with every cell in the QUD. If we implement content calibration in a unsophisticated way, we will make the calibrated content of (¬C) utterly uninformative, as represented in (16).

(15) Semantic content of (C) (blue cross):

180+/-5	200+/-5
190+/-5 x	210+/-5

(16) Unsophisticated calibrated content for (C):

180+/-5	200+/-5
190+/-5 x	210+/-5

So the sophisticated pointist would not say that we calibrate the content of (C) by directly applying its semantic content to the QUD. Instead, they would derive the calibrated content of (¬C) recursively by first deriving the calibrated content of (C), and then subtracting it from the context set:

(17) Calibrated content of (C):

180+/-5	200+/-5
190+/-5 x	210+/-5

(18) Sophisticated calibrated content of (¬C):

180+/-5	200+/-5
190+/-5 x	210+/-5

So the pointist has the resources to deliver intuitively correct results for negations.

2.3.2 'Exactly'

A common move made by the pointists is to have 'exactly' modifies the QUD by making it maximally intolerant of small differences. For example, consider:

(EC) Curry is exactly 190 cm

Its content will be coarsened against the QUD *What is Curry's absolutely exact height?* rather than the QUD we considered above, i.e. *What is Curry's height (to the nearest 5 cm)?*, and so its coarsened content is identical to its (strict) semantic content.

3 Argument against pointism from implicature

We will now take the perspective of the intervalist and consider an argument against the pointist.

(19) **Intervalism's commitments:**

a. '190 cm' denotes an interval, and so (C) can be true if Curry's height falls into that interval.

b. Since '190 cm' denotes an interval, 'exactly' serves to shrink that interval (instead of being vacuous)

3.1 The basic idea of the argument

Consider:

(¬EC) Curry is not exactly 190 cm

(¬C) Curry is not 190 cm.

We will follow Wikipedia and assume that Curry's actual height is 6'3 (190.5 cm). We can observe that while (¬EC) sounds both true and apt, (¬C) at least sounds a bit pedantic, and it may mislead the hearer (if they are ignorant of Curry's height) into thinking that Curry's height is not close to 190 cm. Here is a hypothesis that explains the contrast:

(H) (¬EC) implicates that Curry's height is close to 190 cm, but (¬C) doesn't carry such implicature.

Let's call the inference from (¬EC) that Curry is close to 190 cm a *just-miss inference*. **Does pointism have the resources to account for the inference?**

Notice that, according to pointism, (EC) and (C) have the same semantic content, which means that their negations (¬EC) and (¬C) also have the same semantic content:

(EC) Curry is exactly 190 cm.

(C) Curry is 190 cm.

(¬EC) Curry is not exactly 190 cm.

(¬C) Curry is not 190 cm.

How can (¬EC) and (¬C) mean different things despite having the same semantic content? The following line of explanation doesn't seem to work:

(20) a. Peter caused the car to stop.

b. Peter stopped the car.

The M-principle 'Marked or more prolix expressions warn of an abnormal situation' (Levinson 2000) can explain why while both (20-a) and (20-b) say the same thing, only (20-a) means that Peter stopped the car in an abnormal way. But the M-principle doesn't seem to help the pointist account for the just-miss inference because being close to 190 cm is not an abnormal way of not being 190 cm.

3.2 Intervalist account of just-miss inference

According to the intervalist, '190 cm' denotes an interval of points, and 'exactly' serves to narrow the interval denoted by '190 cm'. Here is a graphical representation of the semantic contents (in red) of (EC) and (C) under intervalism:

(EC) | 186+/-1 | 188+/- 1 | 190+/- 1 | 192+/- 1 | 194+/- 1 | i.e. height(Curry) $\in [189, 191]$

(C) | <——- 190+/- 5 ——-> | i.e. height(Curry) $\in [185, 195]$

The idea is that (EC) places Curry's height within a narrower interval than (C) does. This means that the semantic content of (EC) is stronger than that of (C), and so **the semantic content of (¬EC) is weaker than that of (¬C)** :

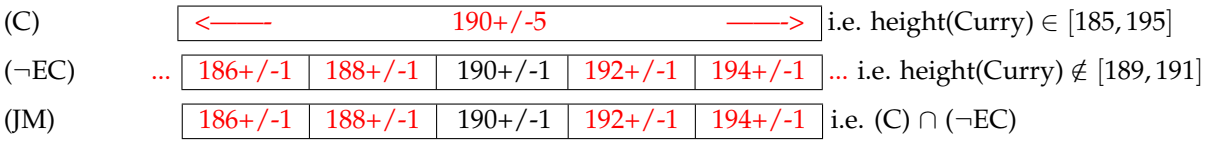
(¬EC) ... | 186+/-1 | 188+/-1 | 190+/-1 | 192+/-1 | 194+/-1 | ... i.e. height(Curry) $\notin [189, 191]$

(¬C) ... | <——- 190+/- 5 ——> | ... i.e. height(Curry) $\notin [185, 195]$

Notice as well that **(¬EC) is not only weaker than (¬C) but also more prolix**. So if a speaker utters (¬EC), the hearer will ask why they do not utter the simpler and more informative (¬C) instead. A plausible explanation would have the same structure as the Gricean explanation of scalar implicature:

(21) **Gricean explanation**:
Suppose the speaker is well informed about Curry's height. The reason why they do not utter the simpler and more informative (¬C) is that they think it is too strong (and hence false). So the hearer reasons that Curry's height doesn't fall outside of the interval [185,195] and hence that Curry's height falls within the interval [185,195].

Here is a graphical representation of the explanation:



So the hearer concludes that Curry's height falls into the gappy interval [185,189]∪[191,195], that is, his height is close to 190 cm.

3.3 Further evidence that just-miss inference is a scalar implicature

Just-miss inferences hold when 'not-exactly' appears in upward-entailing environments but disappear when 'not-exactly' appears in downward-entailing environments, which makes the sentence with 'exactly' stronger than its exactly-free alternative. This is a sign that they are scalar implicatures. Consider:

'Not-exactly' in upward-entailing environments:

(22) Some/ many steel bars aren't exactly 10 m.
⇝ Some/ many steel bars are close to 10m.

(23) Explanation for (22):
a. Some/ many steel bars aren't exactly 10 m < (i.e. weaker than)
b. Some/ many steel bars aren't 10 m

(24) Every handmade chair that is advertised to be 50cm wide isn't exactly 50 cm wide. (Some are off by as much as 8 cm)
⇝ Some chair that is advertised to be 50 cm wide is close to 50 cm wide.

(25) Explanation for (24):
a. Every handmade chair that is advertised to be 50cm wide isn't exactly 50 cm wide. < (i.e. weaker than)
b. Every handmade chair that is advertised to be 50cm wide isn't 50 cm wide.

'Not exactly' in downward-entailing environments:

(26) Every chair that isn't exactly 50cm wide has to be remade.
↛ Some chair that isn't 50 cm wide doesn't have to be remade.

(27) Explanation for (26):
a. Every chair that isn't exactly 50 cm wide has to be remade >
b. Every chair that isn't 50 cm has to be remade.

(28) If this chair isn't exactly 50cm wide, it has to be remade.
↛ It isn't true that if this chair isn't 50cm wide, it has to be remade.

(29) Explanation for (28):
a. If this chair isn't exactly 50cm wide, it has to be remade. >
b. If this chair isn't 50 cm wide, it has to be remade.

4 Towards an intervalist account that can avoid arbitrary selection of intervals and other problems

4.1 Arbitrariness

- Recall that the two motivations of pointism are that (i) it avoids ambiguity and that (ii) it avoids arbitrary selection of intervals.
- But intervalism is not guilty of (i) because it only says that '190 cm' denotes different intervals in different contexts, and context-dependence isn't ambiguity — for example, that 'I' and 'tall' are

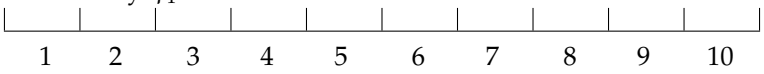
context-dependent doesn't mean that they are ambiguous.
- So to show that intervalism can satisfy the pointist's motivations, we need to tackle (ii).

(30) **Proposal:**

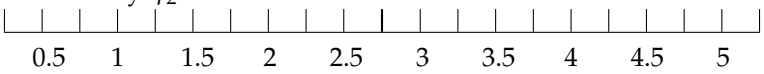
a. Instead of assuming that the context set is a set of possible worlds, we assume that the context set is a set of possible world-granularity pair. (See Barker 2002 for a similar treatment of vagueness)
b. A **granularity** is a function that:
(a) maps a point (e.g. 190 cm) to an interval containing it and
(b) partitions the scale of height into equally wide intervals.

For example, the followings are two real number segments under different granularities:

(31) Granularity γ_1:



(32) Granularity γ_2:



The idea is that under γ_1, the points in the interval (0.5 cm,1.5 cm) are all indistinguishable from 1 cm, and they are all mapped to 1 cm; under γ_2, the points in the interval (0.25 cm,0.75 cm) are all indistinguishable from 0.5 cm, and they are all mapped to 0.5 cm.

(33) **Proposal (cont.):**
The meaning of (C) is a function from context set to context set, such that given a context set, (C) eliminates from it the world-granularity pairs that do not satisfy it.

(34) Context before acceptance of 'Curry is 190 cm':

Curry's height in w	$\gamma(190cm)$
190.5 cm	[190 cm,190 cm]
190.5 cm	[189 cm,191 cm]
191 cm	[189.5 cm,190.5 cm]
191 cm	[185 cm,195 cm]

(35) Context after acceptance of 'Curry is 190 cm':

Curry's height in w	$\gamma(190cm)$
190.5 cm	[190 cm,190 cm]
190.5 cm	[189 cm,191 cm]
191 cm	[189.5 cm,190.5 cm]
191 cm	[185 cm,195 cm]

Since all the world-granularity pairs that fail to satisfy the sentence will be eliminated, and all the world-granularity pairs that satisfy the sentence will survive the acceptance of (C), there is no need to arbitrarily assign an interval to an utterance or a context of utterance.

4.2 Multiple numerical expressions in a sentence

But I don't think that we need only one granularity function to specify a possibility because when there are multiple numerical expressions in the same sentence (or the same text), they can be interpreted based on different granularities/ margins of error:

(36) **66 million years old**, **4 metres high**, **12 metres long** and with almost all its terrifying teeth intact: Tristan Otto, the king of the dinosaurs, is in Copenhagen. (https://snm.ku.dk/english/exhibitions/tristan_otto/)

(37) a. Q: How old are these exhibits?
b. A: This painting, dated 1921, is exactly 100 years old, and the dinosaur over there is 2 billion years old.

So ultimately, we need to specify a possibility in a context set using a world and an **infinite sequence of granularities**, so that the the n-th numerical expression in a text will be interpreted based on the n-th granularity in the sequence.

4.3 '190 cm', 'exactly 190 cm' and 'about 190 cm': How are they different

Here for simplicity we assume that in every sentence there is only one numerical expression, and there is only one granularity in a possibility. (Eventually, we need to drop this assumption and combine the proposal below with there being an infinite sequence of granularities in a possibility.)

(38) **The issues:**

a. 'Exactly 1000 miles' in (3) (repeated below as (39)) doesn't mean exactly 1000 miles (to the nearest nanometer). It is also interpreted imprecisely.

b. A promising intervalist account due to Sauerland and Stateva (2011) makes '190 cm' equivalent to 'about 190 cm'.

(39) Tampa is **exactly 1000 miles** away from New York city.

Consider:

(C) Curry is 190 cm

(EC) Curry is exactly 190 cm

(AC) Curry is about 190 cm

(40) **Proposal:**

a. (C) lets the world-granularity pairs (in the context set) that satisfy it survive.

b. (EC) first eliminates all the world-granularity pairs whose granularity isn't the most finegrained, and then lets the world-granularity pairs that satisfy (C) survive.

c. (AC) first eliminates all the world-granularity pairs whose granularity isn't the least finegrained, and then lets the world-granularity pairs that satisfy (C) survive.

References

Barker, C. (2002). The Dynamics of Vagueness. *Linguistics and Philosophy*, 25(1):1–36.

Carter, S. (2017). Loose talk, negation and commutativity: A hybrid static-dynamic theory. In *Proceedings of Sinn und Bedeutung 21*, pages 267–282. https://ojs.ub.uni-konstanz.de/sub/index.php/sub/article/view/137.

Carter, S. (2021). The dynamics of loose talk. *Noûs*, 55(1):171–198.

Hoek, D. (2018). Conversational exculpature. *Philosophical Review*, 127(2):151–196.

Klecha, P. (2018). On unidirectionality in precisification. *Linguistics and Philosophy*, 41(1):87–124.

Lasersohn, P. (1999). Pragmatic Halos. *Language*, pages 522–551.

Levinson, S. C. (2000). *Presumptive Meanings: The Theory of Generalized Conversational Implicature*. MIT Press.

Roberts, C. (2012). Information structure: Towards an integrated formal theory of pragmatics. *Semantics and Pragmatics*, 5:1–69.

Sauerland, U. and Stateva, P. (2011). Two Types of Vagueness. In *Vagueness and Language Use*, pages 121–145. Springer.

Stalnaker, R. (1978). Assertion. Reprinted in *Context and Content: Essays on Intentionality in Speech and Thought*. Oxford University Press.