To move or not to move

Some consequences of a logical analysis of the English auxiliary system

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(joint work with Robert Levine)

National Institute for Japanese Language and Linguistics

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My starting point

Preface to the second edition (1999):

Why should an introduction to syntax consider more than one framework? One reason is the way the field is. There is no generally accepted theoretical framework. ...
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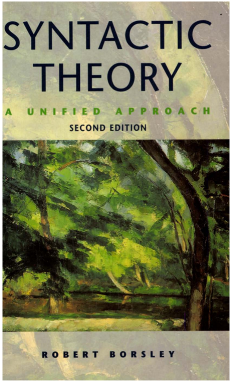

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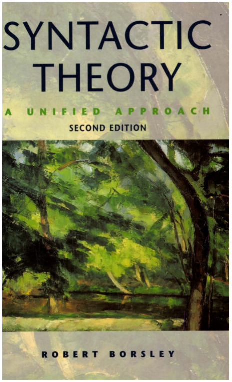

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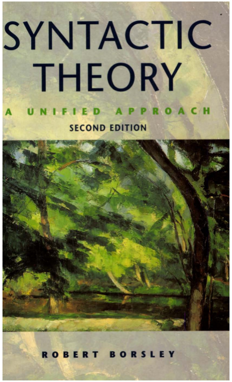
Naïve student: But I want to know the truth... ☹



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☺: ...ideas that will turn out to be of lasting importance – ☹: But which one?

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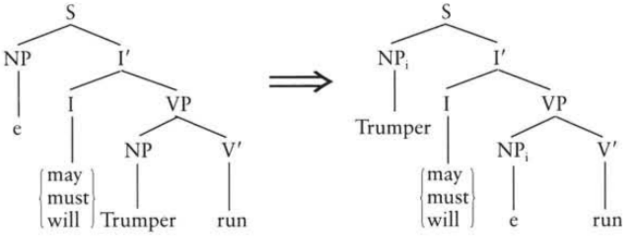
Transformational analysis:

(72)



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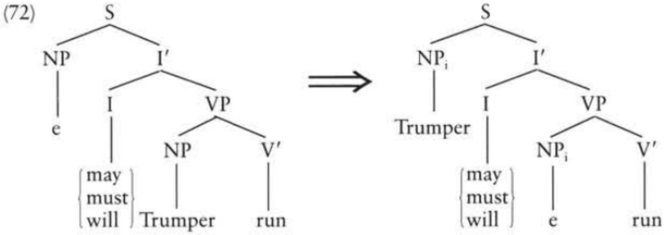
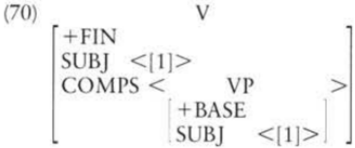
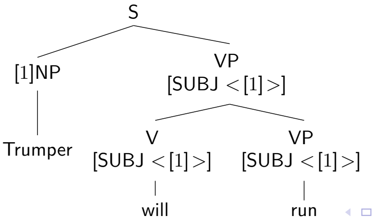


Nontransformational analysis:





Today's plan:

- Revisit the 'to move or not to move' question
- Domain: the English auxiliary system
  - starting point for TG (Chomsky 1957)
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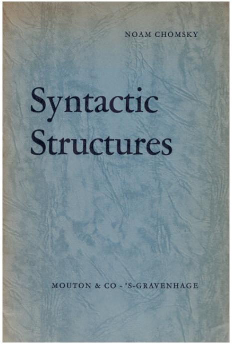

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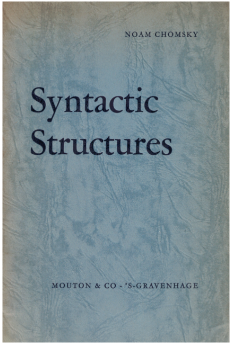

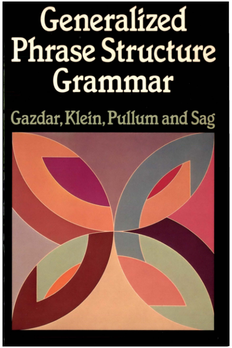

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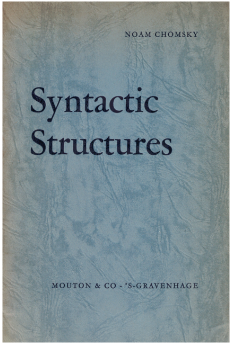

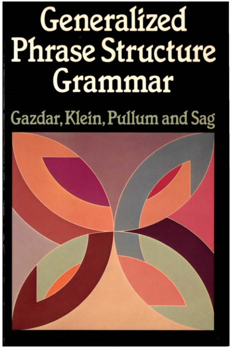

The English auxiliary system (EAS) – the NICE properties

(1) John $\left\{\begin{array}{c}\text{will}\\ \text{should}\\ \text{can}\end{array}\right\}$ buy the book.

(2) **N:** John $\left\{\begin{array}{c}\text{will}\\ \text{should}\\ \text{can}\end{array}\right\}$ not buy the book. (cf. *John buys not the book.)

I: $\left\{\begin{array}{c}\text{Will}\\ \text{Should}\\ \text{Can}\end{array}\right\}$ John buy the book? (cf. *Buys John the book?)

C: John $\left\{\begin{array}{c}\text{won't}\\ \text{shouldn't}\\ \text{can't}\end{array}\right\}$ buy the book. (cf. *John buysn't the book.)

E: Who will buy the book? – John $\left\{\begin{array}{c}\text{will}\\ \text{should}\\ \text{can}\end{array}\right\}$. (cf. *John buys.)

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The English auxiliary system (EAS) – unstresed *do*

(4) **N:** John $\left\{\begin{array}{c}\text{dĭd}\\ \text{dŏes}\end{array}\right\}$ not buy the book.

I: $\left\{\begin{array}{c}\text{Dĭd}\\ \text{Dŏes}\end{array}\right\}$ John buy the book?

C: Who $\left\{\begin{array}{c}\text{bought}\\ \text{buys}\end{array}\right\}$ the book? – John $\left\{\begin{array}{c}\text{dĭd}\\ \text{dŏes}\end{array}\right\}$.

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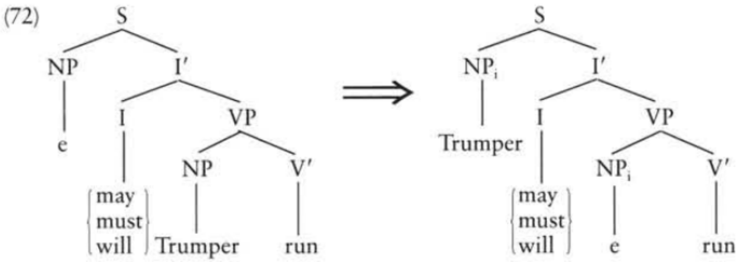
EAS in transformational syntax

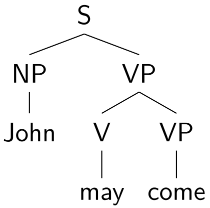


- Modal auxiliaries are raising verbs.
- They semantically take propositions as arguments.

EAS in nontransformational syntax

- How can we do this if auxiliaries are VP-taking verbs?

(5)



(6) may; $\lambda P \lambda x.\Diamond P(x)$; $\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}$

(7)

$$\dfrac{\begin{array}{l}\text{john;}\\ \mathbf{j};\ \mathrm{NP}\end{array} \quad \dfrac{\begin{array}{l}\text{may;}\\ \lambda P \lambda x.\Diamond P(x);\ \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}\end{array} \quad \begin{array}{l}\text{come;}\\ \mathbf{come};\ \mathrm{VP}_{\mathit{bse}}\end{array}}{\begin{array}{l}\text{may} \bullet \text{come;}\\ \lambda x.\Diamond \mathbf{come}(x);\ \mathrm{VP}_{\mathit{fin}}\end{array}}/\mathrm{E}}{\begin{array}{l}\text{john} \bullet \text{may} \bullet \text{come;}\\ \Diamond \mathbf{come}(\mathbf{j});\ \mathrm{S}_{\mathit{fin}}\end{array}}\backslash\mathrm{E}$$

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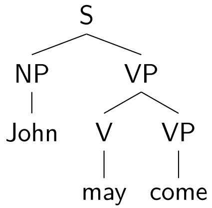

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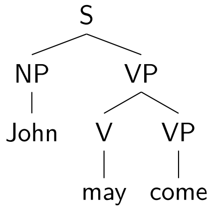

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Some complications for the VP/VP analysis: subject position quantifier

(8) Every student can vote. $(\Diamond > \forall)$

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Standard solution (Bach 1980, Gazdar et al. 1985): lexical type-lifting

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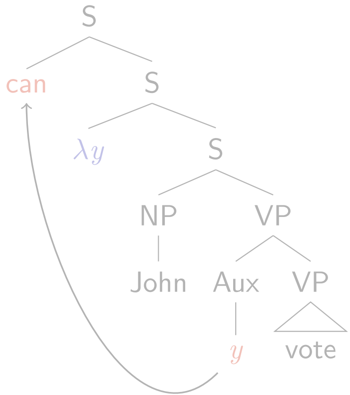
A 'hybrid' analysis of auxiliaries

- Everybody agrees:
  - auxiliaries take scope over a proposition (semantically),
  - but they surface in the preverbal position.
- Here's one way to implement this:

(11)



Cf.:

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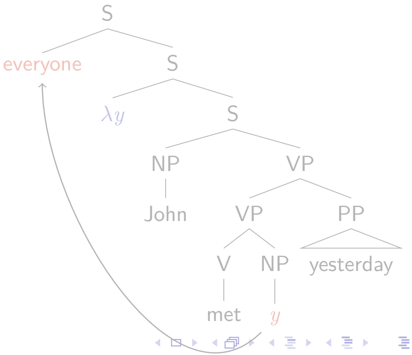

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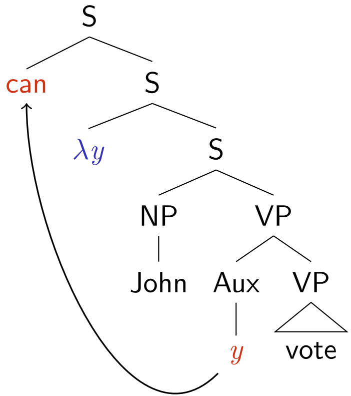

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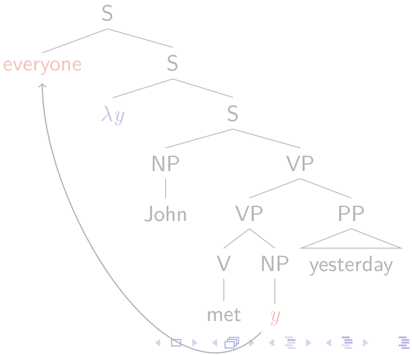

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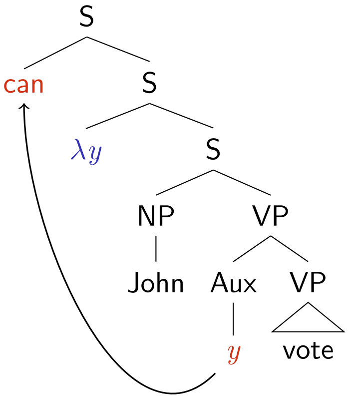

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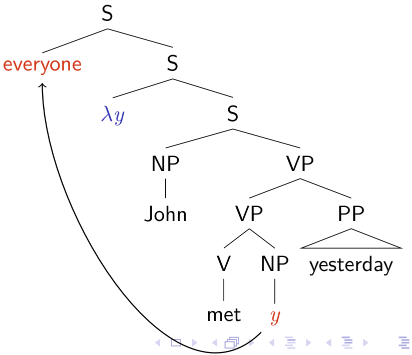

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(13) $\lambda\sigma.\sigma(\mathsf{can})$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et})$; $\mathsf{S}_{fin}\upharpoonright(\mathsf{S}_{fin}\upharpoonright(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$

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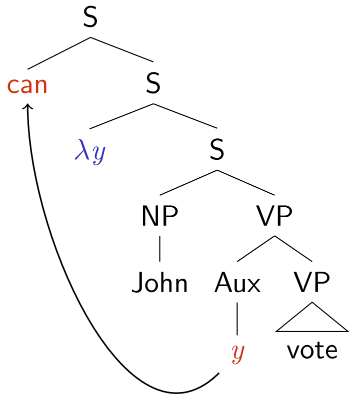

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(14)

$$
\text{②}\!\to\ \dfrac{
\begin{array}{l}\lambda\sigma.\sigma(\text{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et});\\ \text{S}_{fin}\upharpoonright(\text{S}_{fin}\upharpoonright(\text{VP}_{fin}/\text{VP}_{bse}))\end{array}
\quad \text{①}\!\to
\dfrac{\dfrac{\begin{array}{l}\text{john};\\ \mathbf{j};\ \text{NP}\end{array} \quad \dfrac{\left[\begin{array}{l}\varphi_1;\\ f;\ \text{VP}_{fin}/\text{VP}_{bse}\end{array}\right]^1 \quad \begin{array}{l}\text{swim};\\ \mathbf{swim};\ \text{VP}_{bse}\end{array}}{\varphi_1\bullet\text{swim};\ f(\mathbf{swim});\ \text{VP}_{fin}}/\text{E}}{\text{john}\bullet\varphi_1\bullet\text{swim};\ f(\mathbf{swim})(\mathbf{j});\ \text{S}_{fin}}\backslash\text{E}}{\begin{array}{l}\lambda\varphi_1.\text{john}\bullet\varphi_1\bullet\text{swim};\\ \lambda f.f(\mathbf{swim})(\mathbf{j});\ \text{S}_{fin}\upharpoonright(\text{VP}_{fin}/\text{VP}_{bse})\end{array}}\upharpoonright\text{I}^1
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$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

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$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\text{john}\bullet\text{can}\bullet\text{swim};\ \Diamond\mathbf{swim}(\mathbf{j});\ \text{S}_{fin}$$

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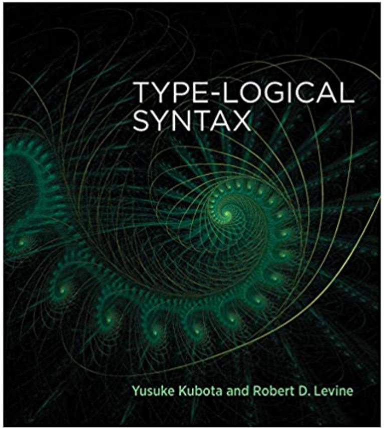
Please read this book for details:



Immediate consequences (1): subject quantifiers

(8) Every student can vote. $(\Diamond > \forall)$

(15)

$$\text{vote};\ \textbf{vote};\ \mathrm{VP}_{\mathit{bse}} \qquad \left[\varphi_1;\ f;\ \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}\right]^1$$

$$\varphi_1 \bullet \text{vote};\ f(\textbf{vote});\ \mathrm{VP}_{\mathit{fin}} \qquad \left[\varphi_2;\ y;\ \mathrm{NP}\right]^2$$

$$\varphi_2 \bullet \varphi_1 \bullet \text{vote};\ f(\textbf{vote})(y);\ \mathrm{S}_{\mathit{fin}}$$

$$\lambda\varphi_2.\varphi_2 \bullet \varphi_1 \bullet \text{vote};\ \lambda y.f(\textbf{vote})(y);\ \mathrm{S}_{\mathit{fin}}{\upharpoonright}\mathrm{NP} \qquad \begin{array}{c}\vdots\\ \lambda\sigma_1.\sigma_1(\text{every} \bullet \text{student});\ \mathbf{V}_{\mathbf{student}};\ \mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{S}_{\mathit{fin}}{\upharpoonright}\mathrm{NP})\end{array}$$

$$\text{every} \bullet \text{student} \bullet \varphi_1 \bullet \text{vote};\ \mathbf{V}_{\mathbf{student}}(\lambda y.f(\textbf{vote})(y));\ \mathrm{S}_{\mathit{fin}}$$

$$\lambda\varphi_1.\text{every} \bullet \text{student} \bullet \varphi_1 \bullet \text{vote};\ \lambda f.\mathbf{V}_{\mathbf{student}}(\lambda y.f(\textbf{vote})(y));\ \mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}) \qquad \lambda\sigma_2.\sigma_2(\text{can});\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et});\ \mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}))$$

$$\text{every} \bullet \text{student} \bullet \text{can} \bullet \text{vote};\ \Diamond\mathbf{V}_{\mathbf{student}}(\lambda y.\textbf{vote}(y));\ \mathrm{S}_{\mathit{fin}}$$

Immediate consequences (1): subject quantifiers

(8) Every student can vote. $(\Diamond > \forall)$

(15)

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \begin{array}{l}\text{vote};\\ \textbf{vote};\\ \mathrm{VP}_{bse}\end{array}
          \quad
          \left[\begin{array}{l}\varphi_1;\\ f;\\ \mathrm{VP}_{fin}/\mathrm{VP}_{bse}\end{array}\right]^1
        }{
          \varphi_1 \bullet \text{vote};\ f(\textbf{vote});\ \mathrm{VP}_{fin}
        }
        \quad
        \left[\begin{array}{l}\varphi_2;\\ y;\\ \mathrm{NP}\end{array}\right]^2
      }{
        \varphi_2 \bullet \varphi_1 \bullet \text{vote};\ f(\textbf{vote})(y);\ \mathrm{S}_{fin}
      }
    }{
      \begin{array}{l}\lambda\varphi_2.\varphi_2 \bullet \varphi_1 \bullet \text{vote};\\ \lambda y.f(\textbf{vote})(y);\ \mathrm{S}_{fin}\upharpoonright\mathrm{NP}\end{array}
    }
    \quad
    \begin{array}{c}\vdots\\ \begin{array}{l}\lambda\sigma_1.\sigma_1(\text{every} \bullet \text{student});\\ \mathbf{V}_{\textbf{student}};\\ \mathrm{S}_{fin}\upharpoonright(\mathrm{S}_{fin}\upharpoonright\mathrm{NP})\end{array}\end{array}
  }{
    \dfrac{
      \begin{array}{l}\text{every} \bullet \text{student} \bullet \varphi_1 \bullet \text{vote};\\ \mathbf{V}_{\textbf{student}}(\lambda y.f(\textbf{vote})(y));\ \mathrm{S}_{fin}\end{array}
    }{
      \begin{array}{l}\lambda\varphi_1.\text{every} \bullet \text{student} \bullet \varphi_1 \bullet \text{vote};\\ \lambda f.\mathbf{V}_{\textbf{student}}(\lambda y.f(\textbf{vote})(y));\ \mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse})\end{array}
    }
  }
  \quad
  \begin{array}{l}\lambda\sigma_2.\sigma_2(\text{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et});\\ \mathrm{S}_{fin}\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))\end{array}
}{
  \text{every} \bullet \text{student} \bullet \text{can} \bullet \text{vote};\ \Diamond\mathbf{V}_{\textbf{student}}(\lambda y.\textbf{vote}(y));\ \mathrm{S}_{fin}
}
$$

Immediate consequences (2): the VP/VP entry is a theorem

(16) $\mathsf{S}_{fin}{\upharpoonright}(\mathsf{S}_{fin}{\upharpoonright}(\mathsf{VP}_{fin}/\mathsf{VP}_{bse})) \vdash \mathsf{VP}_{fin}/\mathsf{VP}_{bse}$

(17)

$$\dfrac{\dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\text{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\text{id}_{et});\\ \mathsf{S}_{fin}{\upharpoonright}(\mathsf{S}_{fin}{\upharpoonright}(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))\end{array} \quad \dfrac{\dfrac{[\varphi_1; x; \mathsf{NP}]^1 \quad \dfrac{[\varphi_2; g; \mathsf{VP}_{fin}/\mathsf{VP}_{bse}]^2 \quad [\varphi_3; f; \mathsf{VP}_{bse}]^3}{\varphi_2 \bullet \varphi_3;\ g(f);\ \mathsf{VP}_{fin}}{/\mathrm{E}}}{\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ g(f)(x);\ \mathsf{S}_{fin}}{\backslash\mathrm{E}}}{\lambda\varphi_2.\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ \lambda g.g(f)(x);\ \mathsf{S}_{fin}{\upharpoonright}(\mathsf{VP}_{fin}/\mathsf{VP}_{bse})}{\upharpoonright\mathrm{I}^2}}{\varphi_1 \bullet \text{can't} \bullet \varphi_3;\ \neg\Diamond f(x);\ \mathsf{S}_{fin}}{\upharpoonright\mathrm{E}}}{\text{can't} \bullet \varphi_3;\ \lambda x.\neg\Diamond f(x);\ \mathsf{VP}_{fin}}{\backslash\mathrm{I}^1}}{\text{can't};\ \lambda f\lambda x.\neg\Diamond f(x);\ \mathsf{VP}_{fin}/\mathsf{VP}_{bse}}{/\mathrm{I}^3}$$

Immediate consequences (2): the VP/VP entry is a theorem

(16) $\mathsf{S}_{fin}\upharpoonright(\mathsf{S}_{fin}\upharpoonright(\mathsf{VP}_{fin}/\mathsf{VP}_{bse})) \vdash \mathsf{VP}_{fin}/\mathsf{VP}_{bse}$

(17)

$$
\dfrac{
  \dfrac{
    \dfrac{
      \boxed{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\mathsf{id}_{et});\\ \mathsf{S}_{fin}\upharpoonright(\mathsf{S}_{fin}\upharpoonright(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))\end{array}}
      \quad
      \dfrac{
        \dfrac{
          [\varphi_1; x; \mathsf{NP}]^1
          \quad
          \dfrac{[\varphi_2; g; \mathsf{VP}_{fin}/\mathsf{VP}_{bse}]^2 \quad [\varphi_3; f; \mathsf{VP}_{bse}]^3}{\varphi_2 \bullet \varphi_3;\ g(f);\ \mathsf{VP}_{fin}}\,/\mathsf{E}
        }{\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ g(f)(x);\ \mathsf{S}_{fin}}\,\backslash\mathsf{E}
      }{\lambda\varphi_2.\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ \lambda g.g(f)(x);\ \mathsf{S}_{fin}\upharpoonright(\mathsf{VP}_{fin}/\mathsf{VP}_{bse})}\,\upharpoonright\mathsf{I}^2
    }{\varphi_1 \bullet \mathsf{can't} \bullet \varphi_3;\ \neg\Diamond f(x);\ \mathsf{S}_{fin}}\,\upharpoonright\mathsf{E}
  }{\mathsf{can't} \bullet \varphi_3;\ \lambda x.\neg\Diamond f(x);\ \mathsf{VP}_{fin}}\,\backslash\mathsf{I}^1
}{\boxed{\mathsf{can't};\ \lambda f \lambda x.\neg\Diamond f(x);\ \mathsf{VP}_{fin}/\mathsf{VP}_{bse}}}\,/\mathsf{I}^3
$$

Immediate consequences (3): the (GQ\S)/VP entry is also a theorem

(18) $S_{fin}{\upharpoonright}(S_{fin}{\upharpoonright}(VP_{fin}/VP_{bse})) \vdash (GQ\backslash S_{fin})/VP_{bse}$

where $GQ = S_{fin}/(NP\backslash S_{fin})$

(19)

$$
\dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\text{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\mathrm{id}_{et});\\ S_{fin}{\upharpoonright}(S_{fin}{\upharpoonright}(VP_{fin}/VP_{bse}))\end{array} \quad \dfrac{\dfrac{[\varphi_1; \mathscr{F}; GQ]^1 \quad \dfrac{[\varphi_2; g; VP_{fin}/VP_{bse}]^2 \quad [\varphi_3; f; VP_{bse}]^3}{\varphi_2 \bullet \varphi_3;\ g(f);\ VP_{fin}}/E}{\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ \mathscr{F}(g(f));\ S_{fin}}\backslash E}{\lambda\varphi_2.\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ \lambda g.\mathscr{F}(g(f));\ S_{fin}{\upharpoonright}(VP_{fin}/VP_{bse})}{\upharpoonright}I^2}{\dfrac{\dfrac{\varphi_1 \bullet \text{can't} \bullet \varphi_3;\ \neg\Diamond\mathscr{F}(f);\ S_{fin}}{\text{can't} \bullet \varphi_3;\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(f);\ GQ\backslash S_{fin}}\backslash I^1}{\text{can't};\ \lambda f\lambda\mathscr{F}.\neg\Diamond\mathscr{F}(f);\ (GQ\backslash S_{fin})/VP_{bse}}/I^3}{\upharpoonright}E
$$

Immediate consequences (3): the $(\mathrm{GQ}\backslash\mathrm{S})/\mathrm{VP}$ entry is also a theorem

(18) $\mathrm{S}_{fin}\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse})) \vdash (\mathrm{GQ}\backslash\mathrm{S}_{fin})/\mathrm{VP}_{bse}$

where $\mathrm{GQ} = \mathrm{S}_{fin}/(\mathrm{NP}\backslash\mathrm{S}_{fin})$

(19)

$$
\dfrac{\dfrac{\dfrac{\boxed{\begin{array}{l}\lambda\sigma.\sigma(\mathrm{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\mathrm{id}_{et});\\ \mathrm{S}_{fin}\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))\end{array}} \quad \dfrac{\dfrac{[\varphi_1;\mathscr{F};\mathrm{GQ}]^1 \quad \dfrac{[\varphi_2; g; \mathrm{VP}_{fin}/\mathrm{VP}_{bse}]^2 \quad [\varphi_3; f; \mathrm{VP}_{bse}]^3}{\varphi_2 \bullet \varphi_3;\; g(f);\; \mathrm{VP}_{fin}}/\mathrm{E}}{\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\; \mathscr{F}(g(f));\; \mathrm{S}_{fin}}\backslash\mathrm{E}}{\lambda\varphi_2.\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\; \lambda g.\mathscr{F}(g(f));\; \mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse})}\upharpoonright\mathrm{I}^2}{\varphi_1 \bullet \mathrm{can't} \bullet \varphi_3;\; \neg\Diamond\mathscr{F}(f);\; \mathrm{S}_{fin}}\upharpoonright\mathrm{E}}{\mathrm{can't} \bullet \varphi_3;\; \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(f);\; \mathrm{GQ}\backslash\mathrm{S}_{fin}}\backslash\mathrm{I}^1}{\boxed{\mathrm{can't};\; \lambda f\lambda\mathscr{F}.\neg\Diamond\mathscr{F}(f);\; (\mathrm{GQ}\backslash\mathrm{S}_{fin})/\mathrm{VP}_{bse}}}/\mathrm{I}^3
$$

Not so immediate, but NICE consequences

- The auxiliary fills in the 'preverbal gap'.

(20) $\lambda\sigma.\sigma(\text{can}); \lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et}); \text{S}_\alpha\upharpoonright(\text{S}_\alpha\upharpoonright(\text{VP}_{\mathit{fin}}/\text{VP}_{\mathit{bse}}))$ **Lex**

(21) john φ come $\xrightarrow[\textbf{Lex}]{}$ john should come

Not so immediate, but NICE consequences

- The auxiliary fills in the 'preverbal gap'.

(20) $\lambda\sigma.\sigma(\mathsf{can}); \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et}); \mathsf{S}_{\alpha}\restriction(\mathsf{S}_{\alpha}\restriction(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$ **Lex**

(21) john φ come $\xrightarrow[\textsc{Lex}]{}$ john should come

Inversion: 'Move $\varphi_{\mathsf{VP/VP}}$ to the initial position.'

(22) $\lambda\sigma\lambda\varphi.\varphi \bullet \sigma(\epsilon); \lambda\mathscr{F}.\mathscr{F}; (\mathsf{S}_{inv}\restriction(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))\restriction(\mathsf{S}_{fin}\restriction(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$ **Inv**

(23) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come $\xrightarrow[\textsc{Lex}]{}$ should john come

Not so immediate, but NICE consequences

- The auxiliary fills in the 'preverbal gap'.

(20) $\lambda\sigma.\sigma(\mathsf{can}); \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et}); \mathsf{S}_{\alpha}\upharpoonright(\mathsf{S}_{\alpha}\upharpoonright(\mathsf{VP}_{\mathit{fin}}/\mathsf{VP}_{\mathit{bse}}))$ **Lex**

(21) john φ come $\xrightarrow[\textbf{Lex}]{}$ john should come

Ellipsis: 'Replace φ_{VP} with $\varphi_{\mathsf{VP/VP}}$.'

(22) $\lambda\sigma\lambda\varphi.\sigma(\varphi); \lambda\mathscr{G}\lambda f.\mathscr{G}(f(P)); (\mathsf{S}_{\mathit{fin}}\upharpoonright(\mathsf{VP}_{\mathit{fin}}/\mathsf{VP}_{\mathit{bse}}))\upharpoonright(\mathsf{S}_{\mathit{bse}}\upharpoonright\mathsf{VP}_{\mathit{bse}})$ **Ell**

(23) john φ_{VP} $\xrightarrow[\textbf{Ell}]{}$ john $\varphi_{\mathsf{VP/VP}}$ $\xrightarrow[\textbf{Lex}]{}$ john should

Not so immediate, but NICE consequences

- The auxiliary fills in the 'preverbal gap'.

(20) $\lambda\sigma.\sigma(\text{can}); \lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et}); \text{S}_\alpha{\upharpoonright}(\text{S}_\alpha{\upharpoonright}(\text{VP}_{\mathit{fin}}/\text{VP}_{\mathit{bse}}))$ **Lex**

(21) john φ come $\xrightarrow[\textsc{Lex}]{}$ john should come

Negation (as a theorem): 'Insert *not* after $\varphi_{\text{VP/VP}}$.'

(22) $\lambda\sigma\lambda\varphi.\sigma(\varphi \bullet \text{not}); \lambda\mathscr{F}\lambda g.\mathscr{F}(\neg g); (\text{S}_{\mathit{fin}}{\upharpoonright}(\text{VP}_{\mathit{fin}}/\text{VP}_{\mathit{bse}})){\upharpoonright}(\text{S}_{\mathit{fin}}{\upharpoonright}(\text{VP}_{\mathit{fin}}/\text{VP}_{\mathit{bse}}))$ **Neg**

(23) john φ come $\xrightarrow[\textsc{Neg}]{}$ john φ not come $\xrightarrow[\textsc{Lex}]{}$ john should not come

NIE operators

Inversion: 'Move $\varphi_{VP/VP}$ to the initial position.'

(24) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come $\xrightarrow[\textsc{Lex}]{}$ should john come

Ellipsis: 'Replace φ_{VP} with $\varphi_{VP/VP}$.'

(25) john φ_{VP} $\xrightarrow[\textsc{Ell}]{}$ john $\varphi_{VP/VP}$ $\xrightarrow[\textsc{Lex}]{}$ john should

Negation (as a theorem): 'Insert *not* after $\varphi_{VP/VP}$.'

(26) john φ come $\xrightarrow[\textsc{Neg}]{}$ john φ not come $\xrightarrow[\textsc{Lex}]{}$ john should not come

NIE operators

Inversion: 'Move $\varphi_{VP/VP}$ to the initial position.'

(24) john φ come $\xrightarrow[\textsc{Inv}\text{ then }\textsc{Lex}]{}$ should john come

Ellipsis: 'Replace φ_{VP} with $\varphi_{VP/VP}$.'

(25) john φ_{VP} $\xrightarrow[\textsc{Ell}\text{ then }\textsc{Lex}]{}$ john should

Negation (as a theorem): 'Insert *not* after $\varphi_{VP/VP}$.'

(26) john φ come $\xrightarrow[\textsc{Neg}\text{ then }\textsc{Lex}]{}$ john should not come

Doing it in one step: function composition

(27) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come $\xrightarrow[\textsc{Lex}]{}$ should john come

(28) $\lambda\sigma\lambda\varphi.\varphi \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\mathscr{F}$; $(S_{inv}\upharpoonright(VP_{fin}/VP_{bse}))\upharpoonright(S_{fin}\upharpoonright(VP_{fin}/VP_{bse}))$ **Inv**(ersion)

(29) $\lambda\sigma.\sigma(\text{can})$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et})$; $S_\alpha\upharpoonright(S_\alpha\upharpoonright(VP_{fin}/VP_{bse}))$ **Lex**(ical insertion)

(30) john φ come $\xrightarrow[\textsc{Lex}\,\circ\,\textsc{Inv}]{}$ should john come ‘first do **Inv** and then **Lex**’

Theorem (function composition):

(31) a. $A/B \;\circ\; B/C \;=\; A/C$

b. $A\upharpoonright B \;\circ\; B\upharpoonright C \;=\; A\upharpoonright C$

By composing **Lex** and **Inv**, we obtain (proof omitted):

(32) $\textsc{Lex} \circ \textsc{Inv} = \lambda\sigma.\text{can} \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et})$; $S_{inv}\upharpoonright(S_{fin}\upharpoonright(VP_{fin}/VP_{bse}))$

Doing it in one step: function composition

(27) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come $\xrightarrow[\textsc{Lex}]{}$ should john come

(28) $\lambda\sigma\lambda\varphi.\varphi \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\mathscr{F}$; $(\mathsf{S}_{inv}{\upharpoonright}(\mathsf{VP}_{fin}/\mathsf{VP}_{bse})){\upharpoonright}(\mathsf{S}_{fin}{\upharpoonright}(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$ **Inv**(ersion)

(29) $\lambda\sigma.\sigma(\text{can})$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et})$; $\mathsf{S}_{\alpha}{\upharpoonright}(\mathsf{S}_{\alpha}{\upharpoonright}(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$ **Lex**(ical insertion)

(30) john φ come $\xrightarrow[\textsc{Lex} \circ \textsc{Inv}]{}$ should john come ‘first do **Inv** and then **Lex**’

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(31) a. $A/B \circ B/C = A/C$

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By composing **Lex** and **Inv**, we obtain (proof omitted):

(32) $\textsc{Lex} \circ \textsc{Inv} = \lambda\sigma.\text{can} \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et})$; $\mathsf{S}_{inv}{\upharpoonright}(\mathsf{S}_{fin}{\upharpoonright}(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$

Doing it in one step: function composition

(27) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come $\xrightarrow[\textsc{Lex}]{}$ should john come

(28) $\lambda\sigma\lambda\varphi.\varphi \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\mathscr{F}$; $(\mathrm{S}_{inv}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$ **Inv**(ersion)

(29) $\lambda\sigma.\sigma(\mathrm{can})$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et})$; $\mathrm{S}_\alpha\upharpoonright(\mathrm{S}_\alpha\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$ **Lex**(ical insertion)

(30) john φ come $\xrightarrow[\textsc{Lex}\,\circ\,\textsc{Inv}]{}$ should john come 'first do **Inv** and then **Lex**'

Theorem (function composition):

(31) a. $A/B \circ B/C = A/C$

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By composing **Lex** and **Inv**, we obtain (proof omitted):

(32) $\textsc{Lex} \circ \textsc{Inv} = \lambda\sigma.\mathrm{can} \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et})$; $\mathrm{S}_{inv}\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$

Doing it in one step: function composition

(27) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come $\xrightarrow[\textsc{Lex}]{}$ should john come

(28) $\lambda\sigma\lambda\varphi.\varphi \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\mathscr{F}$; $(\mathrm{S}_{inv}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$ **Inv**(ersion)

(29) $\lambda\sigma.\sigma(\mathrm{can})$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et})$; $\mathrm{S}_{\alpha}\upharpoonright(\mathrm{S}_{\alpha}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$ **Lex**(ical insertion)

(30) john φ come $\xrightarrow[\textsc{Lex}\,\circ\,\textsc{Inv}]{}$ should john come ‘first do **Inv** and then **Lex**’

Theorem (function composition):

(31) a. $\mathrm{A}/\mathrm{B} \;\circ\; \mathrm{B}/\mathrm{C} \;=\; \mathrm{A}/\mathrm{C}$

b. $\mathrm{A}\upharpoonright\mathrm{B} \;\circ\; \mathrm{B}\upharpoonright\mathrm{C} \;=\; \mathrm{A}\upharpoonright\mathrm{C}$

By composing **Lex** and **Inv**, we obtain (proof omitted):

(32) $\textsc{Lex} \circ \textsc{Inv} = \lambda\sigma.\mathrm{can} \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et})$; $\mathrm{S}_{inv}\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$

Doing it in one step: function composition

(27) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come $\xrightarrow[\textsc{Lex}]{}$ should john come

(28) $\lambda\sigma\lambda\varphi.\varphi \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\mathscr{F}$; $(\mathrm{S}_{inv}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$ **Inv**(ersion)

(29) $\lambda\sigma.\sigma(\mathrm{can})$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et})$; $\mathrm{S}_{\alpha}\upharpoonright(\mathrm{S}_{\alpha}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$ **Lex**(ical insertion)

(30) john φ come $\xrightarrow[\textsc{Lex} \circ \textsc{Inv}]{}$ should john come ‘first do **Inv** and then **Lex**’

Theorem (function composition):

(31) a. $\mathrm{A}/\mathrm{B} \;\circ\; \mathrm{B}/\mathrm{C} \;=\; \mathrm{A}/\mathrm{C}$

b. $\mathrm{A}\upharpoonright\mathrm{B} \;\circ\; \mathrm{B}\upharpoonright\mathrm{C} \;=\; \mathrm{A}\upharpoonright\mathrm{C}$

By composing **Lex** and **Inv**, we obtain (proof omitted):

(32) **Lex** $\circ$ **Inv** $= \lambda\sigma.\mathrm{can} \bullet \sigma(\epsilon)$; $\lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et})$; $\mathrm{S}_{inv}\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$

Composition of auxiliary lexical entry and NIE operators

NIE auxiliary entries as theorems

(33) $\textsc{Lex} \circ \textsc{Inv} = \lambda\sigma.\mathsf{should} \bullet \sigma(\epsilon);\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathsf{id}_{et});\ \mathsf{S}_{inv}\upharpoonleft(\mathsf{S}_{fin}\upharpoonleft(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$

(34) $\textsc{Lex} \circ \textsc{Ell} = \lambda\sigma.\sigma(\mathsf{should});\ \lambda\mathscr{G}.\Box\mathscr{G}(P);\ \mathsf{S}_{fin}\upharpoonleft(\mathsf{S}_{bse}\upharpoonleft\mathsf{VP}_{bse})$

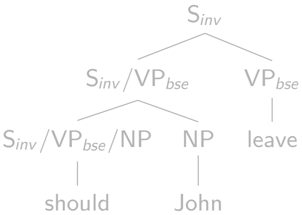
(35) $\textsc{Lex} \circ \textsc{Neg} = \lambda\sigma.\sigma(\mathsf{should} \bullet \mathsf{not});\ \lambda\mathscr{F}.\Box\mathscr{F}(\neg);\ \mathsf{S}_{fin}\upharpoonleft(\mathsf{S}_{fin}\upharpoonleft(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$

Side note (further theorem):

A PSG-style, 'surface-oriented' inverted auxiliary entry:

(36) $\textsc{Lex} \circ \textsc{Inv} \vdash \mathsf{should};\ \lambda x \lambda P.\Box P(x);\ \mathsf{S}_{inv}/\mathsf{VP}_{bse}/\mathsf{NP}$

- Similarly for the other operators (proofs omitted).



Composition of auxiliary lexical entry and NIE operators

NIE auxiliary entries as theorems

(33) $\textsc{Lex} \circ \textsc{Inv} = \lambda\sigma.\text{should} \bullet \sigma(\epsilon);\ \lambda\mathscr{F}.\Box\mathscr{F}(\text{id}_{et});\ \text{S}_{inv}{\upharpoonright}(\text{S}_{fin}{\upharpoonright}(\text{VP}_{fin}/\text{VP}_{bse}))$

(34) $\textsc{Lex} \circ \textsc{Ell} = \lambda\sigma.\sigma(\text{should});\ \lambda\mathscr{G}.\Box\mathscr{G}(P);\ \text{S}_{fin}{\upharpoonright}(\text{S}_{bse}{\upharpoonright}\text{VP}_{bse})$

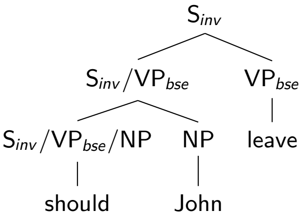
(35) $\textsc{Lex} \circ \textsc{Neg} = \lambda\sigma.\sigma(\text{should} \bullet \text{not});\ \lambda\mathscr{F}.\Box\mathscr{F}(\neg);\ \text{S}_{fin}{\upharpoonright}(\text{S}_{fin}{\upharpoonright}(\text{VP}_{fin}/\text{VP}_{bse}))$

Side note (further theorem):

A PSG-style, 'surface-oriented' inverted auxiliary entry:

(36) $\textsc{Lex} \circ \textsc{Inv} \vdash \text{should};\ \lambda x \lambda P.\Box P(x);\ \text{S}_{inv}/\text{VP}_{bse}/\text{NP}$

- Similarly for the other operators (proofs omitted).



Some more theorems

NIE interactions

(37) a. John will come. **Lex**

b. Will John come? **Lex ∘ Inv**

c. John will ∅. **Lex ∘ Ell**

d. John will not come. **Lex ∘ Neg**

e. Will John not come? **Lex ∘ Inv ∘ Neg**

f. John will not ∅. **Lex ∘ Neg ∘ Ell**

g. Will John? **Lex ∘ Inv ∘ Ell**

h. Will John not ∅? **Lex ∘ Inv ∘ Neg ∘ Ell**

- This essentially follows the insight of the lexical rule-based approach in G/HPSG.

Some more theorems

NIE interactions

(37)
a. John will come. — Lex
b. Will John come? — Lex ∘ Inv
c. John will ∅. — Lex ∘ Ell
d. John will not come. — Lex ∘ Neg
e. Will John not come? — Lex ∘ Inv ∘ Neg
f. John will not ∅. — Lex ∘ Neg ∘ Ell
g. Will John? — Lex ∘ Inv ∘ Ell
h. Will John not ∅? — Lex ∘ Inv ∘ Neg ∘ Ell

- This essentially follows the insight of the lexical rule-based approach in G/HPSG.

But what if there's no modal?

Modal:

(38) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come $\xrightarrow[\textsc{Lex}]{}$ should john come

(39) john φ come $\xrightarrow[\textsc{Lex} \circ \textsc{Inv}]{}$ should john come

Do-less English:

(40) John came.

(41) a. *Came John?
'Is it the case that John came?'

b. *John came not.
'It's not the case that John came.'

(42) john came $\xrightarrow[?]{}$?

(43) john φ come $\xrightarrow[\textsc{Inv}]{}$ φ john come

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But what if there's no modal?

Do-ful English

Suppose we had the following 'phantom' auxiliary that does the same thing as modals:

(44) $\lambda\sigma.\sigma(\text{dı̆d}); \lambda\mathscr{F}.\mathbf{Pst}\,\mathscr{F}(\text{id}_{et}); \text{S}_\alpha\restriction(\text{S}_\alpha\restriction(\text{VP}_{\mathit{fin}}/\text{VP}_{\mathit{bse}}))$ ($\mathbb{LEX}$ 'phantom $\textbf{LEX}$')

Then, we get these ☺:

(45) a. john φ come $\xrightarrow[\mathbb{LEX}\,\circ\,\textbf{NEG}]{}$ john dı̆d not come

b. john φ come $\xrightarrow[\mathbb{LEX}\,\circ\,\textbf{INV}]{}$ dı̆d john come

c. john φ $\xrightarrow[\mathbb{LEX}\,\circ\,\textbf{ELL}]{}$ john dı̆d

Actually, this one too ☹:

(46) john φ come $\xrightarrow[\mathbb{LEX}]{}$ *john dı̆d come

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(45) a. john φ come $\xrightarrow[\mathbb{LEX}\,\circ\,\textbf{Neg}]{}$ john dĭd not come

b. john φ come $\xrightarrow[\mathbb{LEX}\,\circ\,\textbf{Inv}]{}$ dĭd john come

c. john φ $\xrightarrow[\mathbb{LEX}\,\circ\,\textbf{Ell}]{}$ john dĭd

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(46) john φ come $\xrightarrow[\mathbb{LEX}]{}$ *john dĭd come

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Dilemma: With $\mathbb{LEX}$, we get both (47) and (48); without $\mathbb{LEX}$, we get neither. ☹

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b. john φ come $\xrightarrow[\mathbb{LEX}\,\circ\,\textbf{INV}]{}$ dĭd john come

c. john φ $\xrightarrow[\mathbb{LEX}\,\circ\,\textbf{ELL}]{}$ john dĭd

(48) john φ come $\xrightarrow[\mathbb{LEX}]{}$ *john dĭd come

What went wrong?

- $\mathbb{LEX}$ is a phantom auxiliary! It doesn't exist.
- We were fooled by **Do**:

(49) $\textbf{Do}(\textbf{NEG}/\textbf{ELL}/\textbf{INV}) \equiv \mathbb{LEX} \circ \textbf{NEG}/\textbf{ELL}/\textbf{INV}$

(50) john φ come $\xrightarrow[\textbf{Do}(\textbf{INV})]{}$ dĭd john come

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Dilemma: With $\mathbb{LEX}$, we get both (47) and (48); without $\mathbb{LEX}$, we get neither. ☹

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c. john φ $\xrightarrow[\mathbb{L}\mathrm{EX} \circ \textbf{E}\textsc{ll}]{}$ john dı̆d

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Defining Do

With f = **Neg**/**Ell**/**Inv**,

(51) $\textbf{Do}(f) \equiv \mathbb{LEX} \circ f$

So,

(52) $\textbf{Do} = \lambda f.\ \mathbb{LEX} \circ f$
$= \lambda f \lambda x.\ \mathbb{LEX}(f(x))$
$= \lambda\rho\lambda\sigma.\rho(\sigma)(\text{dĭd});\ \lambda\mathscr{G}\lambda h.\mathbf{Pst}\,\mathscr{G}(h)(\text{id}_{et});\ (\text{S}_\beta{\upharpoonright}\text{X}){\upharpoonright}(\text{S}_\alpha{\upharpoonright}(\text{VP}_{fin}/\text{VP}_{bse}){\upharpoonright}\text{X})$

where $\text{X} \in \{\text{S}_{fin}{\upharpoonright}(\text{VP}_{fin}/\text{VP}_{bse}),\ \text{S}_{bse}{\upharpoonright}\text{VP}_{bse},\ \text{S}_{fin}{\upharpoonright}(\text{VP}_{fin}/\text{VP}_{bse})\}$

(53) john φ come $\xrightarrow[\textbf{Do}(\textbf{Inv})]{}$ dĭd john come

- **Do** closes off the VP/VP gap by directly applying to the NIE operators.
- It can't work alone. So, we predict: ☺

(54) *John dĭd buy the book.

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Summary: The logic of *do* insertion

(55) a. john φ come $\xrightarrow[\textsc{Lex}]{}$ john should come

b. john φ come $\xrightarrow[\textsc{Lex} \circ \textsc{Neg}]{}$ john should not come

c. john φ come $\xrightarrow[\textsc{Lex} \circ \textsc{Inv}]{}$ should john come

d. john φ $\xrightarrow[\textsc{Lex} \circ \textsc{Ell}]{}$ john should

(56) a. john came

b. john φ come $\xrightarrow[?]{}$

c. john φ come $\xrightarrow[?]{}$

d. john φ $\xrightarrow[?]{}$

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(56) a. john φ come $\xrightarrow[\mathbb{L}\mathbb{EX}]{}$ *john dĭd come

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Do insertion as a 'last resort' *lexical* operation

- Just as **Lex** $\circ$ **Neg**, etc., can be thought of as an abstract lexical entries, **Do**(**Neg**), etc., can be through of as an abstract lexical entries.

(57)
a. john came

b. john φ come $\xrightarrow[\textbf{Do(Neg)}]{\cancel{\textbf{Lex} \circ \textbf{Neg}}}$ john dĭd not come

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d. john φ $\xrightarrow[\textbf{Do(Ell)}]{\cancel{\textbf{Lex} \circ \textbf{Ell}}}$ john dĭd

- Chomsky (1957) was almost right (but not quite). ☺☹
- Gazdar et al. (1982) were almost right (but not quite). ☺☹
- Everything makes sense if we do it in logic. ☺

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But where did **Do** come from? (Warner 1993)

Stage I (early 16th century)

- Modals are established as a lexical class
- *Do* initially develops as an auxiliary
  - *Do* at this stage has a lexical meaning associated with a range of pragmatic functions
  - Constant rate in both inversion and affirmative contexts

Stage II (from late 16th century onward)

- Steady decline of *do* in affirmative
  - Reanalysis of *do* as a purely tense/aspect auxiliary in interrogative (child learning?)
  - Affirmative *do* declines via blocking
    $\Rightarrow$ *Do* and tense affix as allomorphs

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Adult grammar

(58) a. john φ come $\xrightarrow[\mathbb{L}\text{EX}]{}$ john dĭd come (literary style)

b. john φ come $\xrightarrow[\mathbb{L}\text{EX} \circ \textbf{Inv}]{}$ dĭd john come (colloquial)

- $\mathbb{L}\text{EX}$ is lexically associated with pragmatic focus on truth/polarity.

Where did **Do** come from? (Reinterpreting Warner 1993)

- $\mathbb{LEX}$ used to exist, but it got replaced by **Do**.

Child grammar

(58) a. john φ come $\longrightarrow$ john dĭd come (literary style)

b. john φ come $\xrightarrow[\mathbb{LEX} \circ \textbf{INV}]{}$ dĭd john come (colloquial)

- $\mathbb{LEX} \circ$ **Inv** is reanalyzed as a simple tense auxiliary.
  (Note that polarity happens to be an inherent property of yes/no questions—so, they 'got it wrong' in interpreting adult utterance.)

Where did **Do** come from? (Reinterpreting Warner 1993)

- $\mathbb{L}\text{EX}$ used to exist, but it got replaced by **Do**.

Child grammar

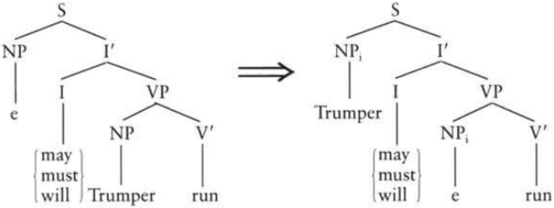
(58) a. john came

b. john φ come $\xrightarrow[\cancel{\mathbb{L}\text{EX} \circ \textbf{INV}}]{\textbf{Do(Inv)}}$ dı̆d john come

- There's not enough evidence to infer that $\mathbb{L}\text{EX}$ is an independent lexeme, so, the most conservative hypothesis given available data is **Do**.

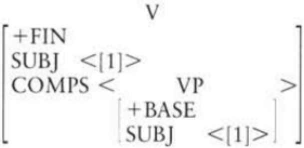
Conclusion

(72)



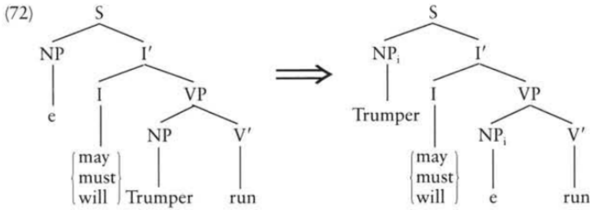
vs.

(70)

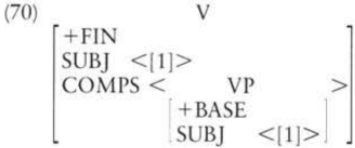


- Looking at the same thing from different angles eventually pays off.

Conclusion



vs.

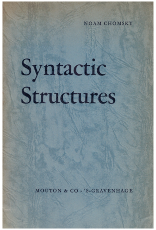


$\Downarrow$

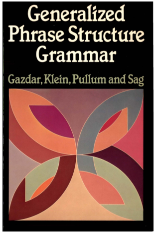
$$\lambda\sigma.\sigma(\mathsf{may});\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et});\ \mathsf{S}_{fin}\upharpoonright(\mathsf{S}_{fin}\upharpoonright(\mathsf{VP}_{fin}/\mathsf{VP}_{bse}))$$

- Looking at the same thing from different angles eventually pays off.
- Integrating the insights of competing approaches can lead to new insights.

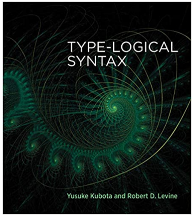
Thanks!



vs.



$\Downarrow$



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References

See the following two sources for references:

- Kubota and Levine (2021, 2020)

N.B.: The analysis of *do* insertion in Kubota and Levine (2021) is somewhat different from the one presented in this talk.

Kubota, Y. and Levine, R. (2020). *Type-Logical Syntax*. MIT Press, Cambridge, MA. Available Open Access at https://direct.mit.edu/books/book/4931/Type-Logical-Syntax.

Kubota, Y. and Levine, R. (2021). NPI licensing and the logic of the syntax-semantics interface. *Linguistic Research*, pages 00–00. Available at https://ling.auf.net/lingbuzz/005918.

Appendix: Overgeneration? (Kubota and Levine, 2020, Section 9.2.2)

(59) $\lambda\sigma.\sigma(\mathsf{should}); \lambda\mathscr{G}.\Box\mathscr{G}(\mathrm{id}_{et}); \mathsf{S}^n_\alpha{\upharpoonright}(\mathsf{S}^n_\alpha{\upharpoonright}(\mathsf{VP}^n_{\mathit{fin}}/\mathsf{VP}^n_{\mathit{bse}}))$

(60)

$$\dfrac{\begin{array}{l}\vdots \\ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \\ \quad \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}); \\ \mathrm{S}^1_{f,+}\end{array} \qquad \begin{array}{l}\mathsf{think}; \\ \mathbf{think}; \\ \mathrm{VP}^{n+1}_{b,\varnothing}/\mathrm{S}^n_{f,+}\end{array}}{\begin{array}{l}\mathsf{think} \circ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \\ \quad \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a})); \mathrm{VP}^2_{b,\varnothing}\end{array}}$$

$$\dfrac{\begin{array}{l}\mathsf{think} \circ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \\ \quad \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a})); \mathrm{VP}^2_{b,\varnothing}\end{array} \qquad \begin{array}{l}\vdots \\ \mathsf{may}; \\ \lambda Q \lambda z. \\ \quad \Diamond Q(z); \\ \mathrm{VP}^2_{f,+}/\mathrm{VP}^2_{b,\varnothing}\end{array}}{\begin{array}{l}\mathsf{may} \circ \mathsf{think} \circ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \lambda z.\Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(z); \mathrm{VP}^2_{f,+}\end{array}}$$

$$\dfrac{\begin{array}{l}\mathsf{may} \circ \mathsf{think} \circ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \lambda z.\Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(z); \mathrm{VP}^2_{f,+}\end{array} \qquad \begin{array}{l}\mathsf{john}; \\ \mathbf{j}; \\ \mathrm{NP}\end{array}}{\begin{array}{l}\mathsf{john} \circ \mathsf{may} \circ \mathsf{think} \circ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(\mathbf{j}); \mathrm{S}^2_{f,+}\end{array}}$$

$$\dfrac{\begin{array}{l}\mathsf{john} \circ \mathsf{may} \circ \mathsf{think} \circ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(\mathbf{j}); \mathrm{S}^2_{f,+}\end{array}}{\begin{array}{l}\lambda\varphi_1.\mathsf{john} \circ \mathsf{may} \circ \mathsf{think} \circ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \lambda f.\Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(\mathbf{j}); \mathrm{S}^2_{f,+}{\upharpoonright}(\mathrm{VP}^1_{b,\varnothing}/\mathrm{VP}^1_{b,\varnothing})\end{array}}{\upharpoonright}\mathrm{I}^1$$

$$\dfrac{\begin{array}{l}\lambda\varphi_1.\mathsf{john} \circ \mathsf{may} \circ \mathsf{think} \circ \mathsf{ann} \circ \mathsf{should} \circ \varphi_1 \circ \mathsf{buy} \circ \mathsf{the} \circ \mathsf{car}; \\ \lambda f.\Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(\mathbf{j}); \mathrm{S}^2_{f,+}{\upharpoonright}(\mathrm{VP}^1_{b,\varnothing}/\mathrm{VP}^1_{b,\varnothing})\end{array} \qquad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{not}); \\ \lambda\mathscr{G}.\neg\mathscr{G}; \\ \mathrm{S}^n_{\gamma,-}{\upharpoonright} \\ \quad (\mathrm{S}^n_{\gamma,\varnothing}{\upharpoonright}(\mathrm{VP}^n_{b,\varnothing}/\mathrm{VP}^n_{b,\varnothing}))\end{array}}{\text{FAIL}}$$